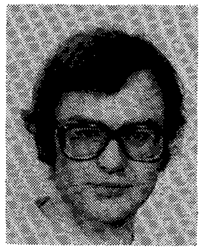
[4] N. D. Megill, "TDR's profile impedances of backplanes and pc boards," *Electronics*, vol. 59, pp. 113–117, July 14, 1981.
[5] A. Papoulis, *The Fourier Integral and its Application*. New York: McGraw-Hill, 1962, pp. 201–210.
[6] J. D. Rhodes, "Fourier coefficient design of stepped impedance transmission line networks," *Int. J. Circuit Theory Appl.*, vol. 1, pp. 363–371, 1973.
[7] G. Saulich, "A new approach in the computation of ultra-high degree equal ripple polynomials for 90°-coupler synthesis," *IEEE Trans. Microwave Theory Tech*, vol. MTT-29, pp. 132–135, Feb. 1981.



Bengt Ulriksson (S'78) was born in Sveg, Sweden, in 1954. He received the M.S. degree in electrical engineering from Chalmers University of Technology, Gothenburg, Sweden in 1977.
In 1977 he joined the Division of Network Theory, Chalmers University of Technology, as a Research and Teaching Assistant, where he has been working with stripline directional couplers and diode phase shifters.

The Use of Symmetry to Simplify the Integral Equation Method with Application to 6-Sided Circulator Resonators

GORDON P. RIBLET, MEMBER, IEEE, AND E. R. BERTIL HANSSON

Abstract—**In this paper it is shown that for planar two-dimensional problems with symmetry, the dimensions of the matrices, which must be inverted to obtain a solution using the integral equation method, can be substantially reduced. For instance, for a three-fold symmetric hexagonal circulator junction with N segments about the periphery, the dimension of matrices to be inverted is reduced to $N/3$ from the usual N. It is demonstrated that for six-sided resonators with three-fold symmetry, a very good approximation to the equivalent admittance can be obtained with only 12 segments around the periphery, meaning that only 4×4 matrices need be inverted.**

I. Introduction

ONE OF THE MOST general methods for analyzing arbitrarily shaped planar circuits is based on the contour integral representation of the wave equation as presented by Okoshi and Miyoshi about 1970 [1], [2]. Initially an isotropic dielectric was assumed. In 1977 the theory was extended to include nonreciprocal circuits which use ferrites magnetized perpendicular to the conducting planes [3]. A drawback of the contour integral method is the relatively long computational times required for analysis since, for complex circuit patterns, large-dimensional matrices must be inverted. Some methods are available for improving the computational efficiency for circuits with special properties. One method is based on the observation that the Green's function suitable to the integral equation method is not unique [4]. The arbitrariness of the Green's function can be used to optimize the accuracy of the numerical results, or equivalently decrease the computational time necessary to obtain a given accuracy. The second method takes advantage of symmetries in the planar circuit. In the design of a 3-dB hybrid with two symmetry planes Okoshi, Imai, and Ito computed the reflection coefficient from one of the four congruent quarter circuits for each of the four eigenexcitations of the entire circuit [5]. The scattering parameters of the hybrid are then given as a linear combination of these scattering matrix eigenvalues. In this paper it will be shown in general how existing symmetries of a junction may be used to simplify the diagonalization of matrices. The computational effort is reduced by a factor comparable with the order of symmetry of the circuit.

Though planar junction, three-port circulators with six-sided resonators constitute a natural application of the integral equation method, and these circulators have been built commercially for many years now, the method has with one exception [6] not been applied to their analysis. Consequently, it was felt to be worthwhile to combine a

Manuscript received December 7, 1981; revised April 1, 1982.
The authors are with the Microwave Development Laboratory, Natick, MA 01760

demonstration of the simplifications resulting from symmetry with a detailed analysis of the frequency dependence of the equivalent admittance of apex-coupled hexagonal-type resonators with different coupling angles. It has been found that with an appropriate choice of the Green's function a very good approximation to the equivalent admittance can be obtained with only 12 segments around the periphery, meaning that only 4×4 matrices need be inverted.

II. Circular Resonators as an Example of the Use of Symmetry to Simplify the Integral Equation Method

The symmetrical three-port circulator with a circular resonator presents a particularly nice application because, in this case, it is possible to use symmetry arguments to obtain a direct solution with the integral equation method, without the need to use a matrix inverse operation. As a result, the integral equation method can be as rapid as analytical methods of obtaining the equivalent admittance [7], [8]. From the integral equation it can be deduced that the electric field E at a point S_0 on the periphery of the planar circuit can be expressed as [3]

$$E(s_0)=2\oint_C\Big[j\omega\mu_{\text{eff}}G(kR)H(s)-k\left(\cos\theta-j\frac{\kappa}{\mu}\sin\theta\right)G'(kR)\cdot E(s)\Big]\,ds. \quad (1)$$

The quantities H and G are the magnetic field and Green's function, respectively, while ω is the frequency, μ_{eff} the effective permeability, μ and κ are the permeability tensor components, and $k=\omega\sqrt{\epsilon\mu_{\text{eff}}}$ where ϵ is the dielectric constant. The quantity R is the length of the vector $\boldsymbol{R}$ connecting the points s_0, s on the periphery while θ is the angle $\boldsymbol{R}$ makes with the normal to the periphery at s. For the numerical evaluation of (1), the contour C can be divided into N segments. If the number of segments is chosen large enough all segments can be made so short that the fields along each segment can be taken as constant. Equation (1) can then be approximated by the following matrix equation:

$$[U]\bar{E}=[T]\bar{H}. \quad (2)$$

The elements of $[U]$ and $[T]$ are given by

$$U_{mn}=2kW_n\left(\cos\theta_{mn}-j\frac{\kappa}{\mu}\sin\theta_{mn}\right)G'(kR_{mn}),\qquad n\neq m \quad (3)$$

$$U_{mm}=1 \quad (4)$$

$$T_{mn}=j2\omega\mu_{\text{eff}}W_nG(kR_{mn}),\qquad n\neq m \quad (5)$$

$$T_{nn}=j2\omega\mu_{\text{eff}}W_n\left[C_0-\frac{1}{2\pi}\left(\ln\frac{kW_n}{2}-1+\gamma\right)\right] \quad (6)$$

where

$$G(kR_{mn})=\left[\tfrac{1}{4}Y_0(kR_{mn})+C_0J_0(kR_{mn})\right]$$
$$G'(kR_{mn})=\left[-\tfrac{1}{4}Y_1(kR_{mn})-C_0J_1(kR_{mn})\right]$$

and where $m,n=1,2,\cdots,N$, W_n is the width of segment n and the angle θ_{mn}, and the distance R_{mn} are related to the center points of segment m and n. In (3)–(6), the general form of the Green's function [14]

$$G=\tfrac{1}{4}Y_0(kR)+\sum_n C_nJ_n(kR)\cos n\theta$$

could be used instead of the simpler form given above which is obtained by setting all coefficients C_n equal to zero except for C_0. The properties of the N-port so formed can be calculated from (2). So, for instance, the wave impedance matrix $[Z]$ is given by [3]

$$[Z]=[U]^{-1}[T]. \quad (7)$$

The wave impedance matrix, or its inverse, the wave admittance matrix, can readily be reduced to the corresponding matrix for the actual multiport, formed by the planar circuit, based on knowledge of either the electric or the magnetic field distribution along the periphery.

From (7) it is evident that the calculation of the impedance matrix of the N-port involves the inversion of an in general complex $N\times N$-matrix. In case the N-port shows some form of symmetry it is often possible to find a way to reduce the computations involved considerably, as in the following case. Consider an N-fold symmetric junction. Assume that we divide the periphery of the junction into N equal segments. The $[U]$- and the $[T]$-matrices then reflect the N-fold symmetry of the junction. So, for instance, the $[U]$-matrix can be written

$$[U]=\begin{bmatrix} U_1 & U_2 & U_3 & \cdots & U_N\\ U_N & U_1 & U_2 & \cdots & U_{n-1}\\ U_{N-1} & U_N & U_1 & \cdots & U_{N-2}\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ U_2 & U_3 & U_4 & \cdots & U_1\end{bmatrix}. \quad (8)$$

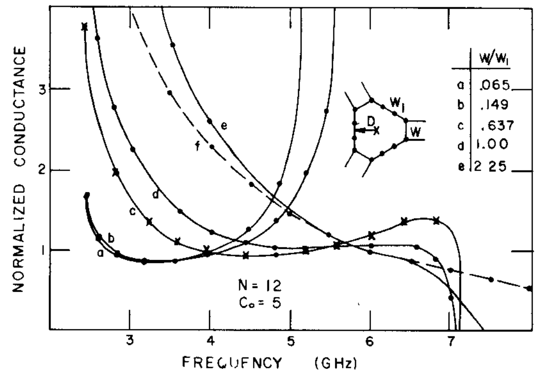
Because of symmetry, there will only be N independent matrix entries U_m. Such a matrix can be manipulated by using the eigenvalue approach. In particular, for such a matrix based on N-fold rotational symmetry the eigenvalues will be known linear functions of the matrix entries and vice versa [9]. Consequently, the matrix $[U]$ can be inverted in the following direct way: 1) express the eigenvalues λ_m, $m=1,\cdots,N$ as a linear function of the matrix entries U_m; 2) determine the scalar inverses $\lambda_m^{-1}=1/\lambda_m$; and 3) express the N-matrix entries U_m^{-1} as a linear function of λ_m^{-1} $m=1,\cdots,N$. Specifically

$$\lambda_m=\sum_{n=1}^{N}U_ne^{j(m-1)(n-1)(2\pi/N)} \quad (9)$$

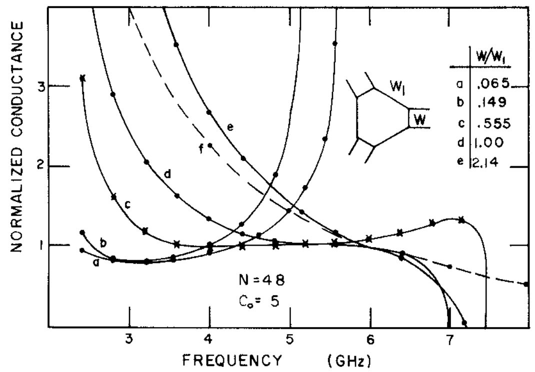
$$U_m^{-1}=\frac{1}{N}\sum_{n=1}^{N}\lambda_n^{-1}e^{-j(m-1)(n-1)(2\pi/N)}. \quad (10)$$

The inverse matrix $[U]^{-1}$ is given in terms of the N-matrix entries U_m^{-1} by the same sort of expression as (8).

As a check on the procedure, a comparison was made between theoretical calculations of the equivalent admittance and experimental results published previously on a

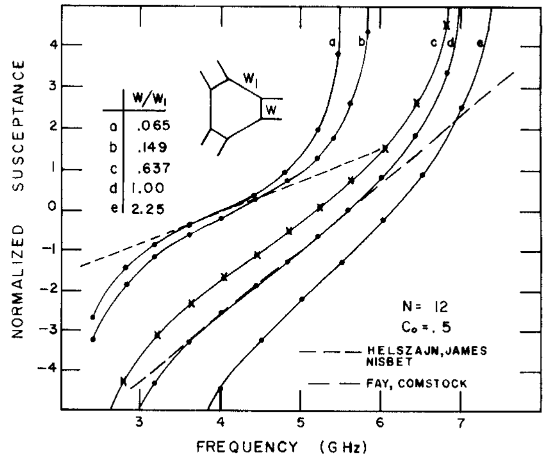


(a)

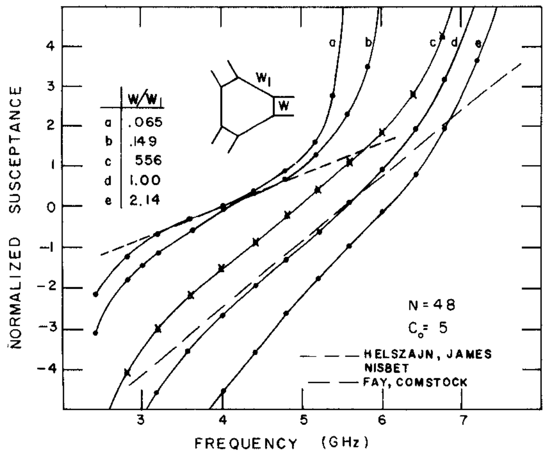


(b)

Fig. 1. The real part of the equivalent admittance of a threefold symmetric hexagonal resonator. The boundary is divided into (a) 12 segments and (b) 48 segments. Design data: $4\pi M_s = 0.08$ T, $4\pi M_r = 0.052$ T, $g_{\text{eff}} = 2.0$, $\Delta H_{3\text{db}} = 4.377$ kat/m, $\epsilon_r = 14.7$, $H_e \to 0+$, $D = 4.06$ mm. $t = 3.94$ mm.



(a)



(b)

Fig. 2. The imaginary part of the equivalent admittance of the resonator in Fig. 1. (a) 12 segments. (b) 48 edge segments.

circular garnet resonator [8]. In the theoretical calculation the tangential field $\overline{H}$ in (2) is, as usual, assumed to be 0 at the periphery, except where the striplines connect to the resonator, at which points it is assumed to be constant. The wave impedance matrix entries $Z_{11}^{(3)}$, $Z_{12}^{(3)}$, and $Z_{13}^{(3)}$ are then calculated on the basis of the average electric field across the striplines

$$Z_{1l}^{(3)} = Z_{1a} + \sum_{k=1}^{p-1} \frac{p-k}{p}\left[Z_{1_{(a+k)}} + Z_{1_{(a-k)}}\right], \qquad l=1,2,3 \tag{11}$$

where $a=(l-1)N/3+1$, and it is assumed that each port occupies p segments. From the wave impedance matrix entries, the equivalent impedance or admittance can be calculated [8],[9]. It should be pointed out that there was essentially no difference in the values computed using the integral equation method and the values computed analytically using Bessel functions, and that the computational times were now similar.

III. The Frequency Dependence of the Equivalent Admittance of 6-Sided Resonators

Although hexagonal-type resonators have been used extensively in planar Y-junction circulators during the last few years, relatively little has been published about the properties of such circulators. In a recent paper Helszajn, James, and Nisbet estimated the Q factor for side and apex coupled triangular circulators to be three times and one-third, respectively, as large as for disk circulators by using a lowest order mode approximation [10]. This result seemed to agree with experiment. However, since the presence of coupling lines causes a disturbance of the field configurations, it can be assumed that these relations apply only for small coupling angles. This section is intended to show, with the aid of various graphs, the properties of hexagonal-type circulators over a wide frequency band centered about the lowest order operating frequency in the low-field limit. The basic resonator is assumed to be 6-sided and three-fold symmetric (see the inset of Fig. 1). The sides with width W connect to the striplines. According to the usual assumption, the tangential magnetic field is assumed to be zero along the sides with width W_1 and constant across the striplines. The adequacy of these assumptions is attested to by the agreement between theory and experiment for circular disk resonators.

Since the circuit in the present case is assumed to have only three-fold symmetry, it is in general not possible to use the arguments of the previous section to invert the $[U]$ matrix directly. However, if there are for instance 12 segments around the periphery (see Figs. 1(a), 2(a)) chosen so that the three-fold symmetry is preserved, then $[U]$ can be written in the form

$$[U]=\left[\begin{array}{cccc|cccc|cccc} U_{11} & U_{12} & U_{13} & U_{14} & U_{15} & U_{16} & U_{17} & U_{18} & U_{51} & U_{52} & U_{53} & U_{54}\\ U_{21} & U_{22} & U_{23} & U_{24} & U_{25} & U_{26} & U_{27} & U_{28} & U_{61} & U_{62} & U_{63} & U_{64}\\ U_{31} & U_{32} & U_{33} & U_{34} & U_{35} & U_{36} & U_{37} & U_{38} & U_{71} & U_{72} & U_{73} & U_{74}\\ U_{41} & U_{42} & U_{43} & U_{44} & U_{45} & U_{46} & U_{47} & U_{48} & U_{81} & U_{82} & U_{83} & U_{84}\\ \hline U_{51} & U_{52} & U_{53} & U_{54} & U_{11} & U_{12} & U_{13} & U_{14} & U_{15} & U_{16} & U_{17} & U_{18}\\ U_{61} & U_{62} & U_{63} & U_{64} & U_{21} & U_{22} & U_{23} & U_{24} & U_{25} & U_{26} & U_{27} & U_{28}\\ U_{71} & U_{72} & U_{73} & U_{74} & U_{31} & U_{32} & U_{33} & U_{34} & U_{35} & U_{36} & U_{37} & U_{38}\\ U_{81} & U_{82} & U_{83} & U_{84} & U_{41} & U_{42} & U_{43} & U_{44} & U_{45} & U_{46} & U_{47} & U_{48}\\ \hline U_{15} & U_{16} & U_{17} & U_{18} & U_{51} & U_{52} & U_{53} & U_{54} & U_{11} & U_{12} & U_{13} & U_{14}\\ U_{25} & U_{26} & U_{27} & U_{28} & U_{61} & U_{62} & U_{63} & U_{64} & U_{21} & U_{22} & U_{23} & U_{24}\\ U_{35} & U_{36} & U_{37} & U_{38} & U_{71} & U_{72} & U_{73} & U_{74} & U_{31} & U_{32} & U_{33} & U_{34}\\ U_{45} & U_{46} & U_{47} & U_{48} & U_{81} & U_{82} & U_{83} & U_{84} & U_{41} & U_{42} & U_{43} & U_{44} \end{array}\right] = \begin{bmatrix} \{U_1\} & \{U_2\} & \{U_3\}\\ \{U_3\} & \{U_1\} & \{U_2\}\\ \{U_2\} & \{U_3\} & \{U_1\} \end{bmatrix} \quad (12)$$

where the square submatrices $\{U_1\}$, $\{U_2\}$, and $\{U_3\}$

$$\{U_1\}=\begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14}\\ U_{21} & U_{22} & U_{23} & U_{24}\\ U_{31} & U_{32} & U_{33} & U_{34}\\ U_{41} & U_{42} & U_{43} & U_{44} \end{bmatrix}$$

$$\{U_2\}=\begin{bmatrix} U_{15} & U_{16} & U_{17} & U_{18}\\ U_{25} & U_{26} & U_{27} & U_{28}\\ U_{35} & U_{36} & U_{37} & U_{38}\\ U_{45} & U_{46} & U_{47} & U_{48} \end{bmatrix}$$

$$\{U_3\}=\begin{bmatrix} U_{51} & U_{52} & U_{53} & U_{54}\\ U_{61} & U_{62} & U_{63} & U_{64}\\ U_{71} & U_{72} & U_{73} & U_{74}\\ U_{81} & U_{82} & U_{83} & U_{84} \end{bmatrix}$$

will be four-dimensional. The 12-dimensional inverse matrix $[U]^{-1}$ will have a similar form with $\{U_1\}$, $\{U_2\}$, and $\{U_3\}$ replaced by four-dimensional matrices $\{U_1^{-1}\}$, $\{U_2^{-1}\}$, and $\{U_3^{-1}\}$. Note that the mathematical form is the same as that for the $[S]$, $[Z]$, or $[Y]$ matrix representations of the symmetrical 3-port circulator. The linear eigenvalue -S matrix entry relations for this device [9] can be used to define four-dimensional eigenmatrices $\{\lambda_1\}, \{\lambda_2\}, \{\lambda_3\}$ given by

$$\{\lambda_1\}=\{U_1\}+\{U_2\}+\{U_3\} \quad (13)$$

$$\{\lambda_2\}=\{U_1\}+\{U_2\}e^{j2\pi/3}+\{U_3\}e^{-j2\pi/3} \quad (14)$$

$$\{\lambda_3\}=\{U_1\}+\{U_2\}e^{-j2\pi/3}+\{U_3\}e^{+j2\pi/3}. \quad (15)$$

Once the inverses of these four-dimensional matrices $\{\lambda_1^{-1}\},\{\lambda_2^{-1}\},\{\lambda_3^{-1}\}$ have been determined, then similar relations may be used to find $\{U_1^{-1}\}$, $\{U_2^{-1}\}$, and $\{U_3^{-1}\}$, i.e.,

$$\{U_1^{-1}\}=\left(\{\lambda_1^{-1}\}+\{\lambda_2^{-1}\}+\{\lambda_3^{-1}\}\right)/3 \quad (16)$$

$$\{U_2^{-1}\}=\left(\{\lambda_1^{-1}\}+\{\lambda_2^{-1}\}e^{-j2\pi/3}+\{\lambda_3^{-1}\}e^{j2\pi/3}\right)/3 \quad (17)$$

$$\{U_3^{-1}\}=\left(\{\lambda_1^{-1}\}+\{\lambda_2^{-1}\}e^{j2\pi/3}+\{\lambda_3^{-1}\}e^{-j2\pi/3}\right)/3. \quad (18)$$

The problem of inverting such a 12-dimensional matrix has thus been reduced to the simpler problem of inverting three four-dimensional matrices by the use of symmetry arguments since

$$[U^{-1}]=\begin{bmatrix} \{U_1^{-1}\} & \{U_2^{-1}\} & \{U_3^{-1}\}\\ \{U_3^{-1}\} & \{U_1^{-1}\} & \{U_2^{-1}\}\\ \{U_2^{-1}\} & \{U_3^{-1}\} & \{U_1^{-1}\} \end{bmatrix}. \quad (19)$$

Figs. 1 and 2 give plots of the normalized conductance and normalized susceptance versus frequency for symmetrical six-sided resonators with various ratios of the stripline width W to the side width $W1$ and biased below resonance. The calculations have been performed with the constant C_0 in the Green's function set to 0.5. This value is different from that chosen by other authors, but we have found it to lead to a rapid convergence for a relatively small number of points around the periphery. This can be seen from the small difference in the curves for $N=12$ and $N=48$ segments about the periphery in Figs. 1 and 2. The segment lengths on any side were taken to be the same, as in the inset of Fig. 1(a). For 12 segments only 4×4 matrices need be inverted. For 4×4 matrices a simple subroutine can be written to perform the inversion without the need to have available the matrix inverse command of Basic, for instance [11]. Consequently, the analysis may be performed on a small desk-top calculator.

It can be shown that the equivalent admittance in the limit of small magnetic fields is given by

$$Y_e(\omega, H)\Big|_{H\simeq 0}=\sqrt{3}\,\frac{\delta Y_1(\omega)}{\delta H}\Big|_{H=0}\cdot H+jY_1(\omega)\big|_{H=0} \quad (20)$$

where H is the applied magnetic field and $Y_1(\omega)$ refers to the eigensusceptance. Consequently, it will be characterized by a susceptance slope parameter and a frequency-dependent conductance in the region where the susceptance is small. The conductance curves have been normalized to 1 at the frequency for which the susceptance is zero, although in fact the calculations were performed for very small magnetic fields of 10^{-4} T. It is apparent from Fig. 3 that the frequency dependence of G changes substantially as the aspect ratio $W/W1$ is varied. For $W/W1 \simeq 0.6$ (the

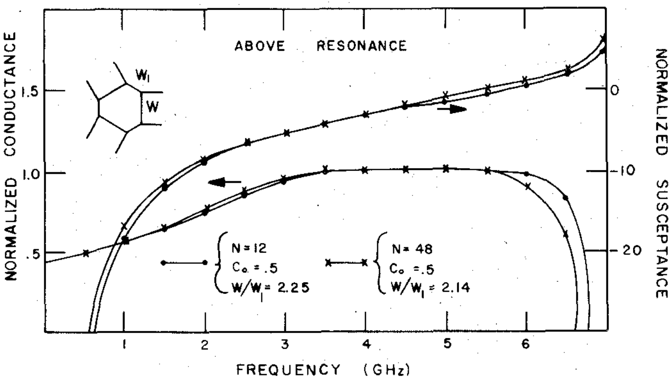


Fig. 3. The equivalent admittance of the resonator in Fig. 1 at the high-field limit ($H_e \to \infty$).

geometry given in the inset figure) it has a broad minimum. In this case, the equivalent admittance can be well approximated by a constant conductance shunted by a pure susceptance—the usual approximation—over more than an octave bandwidth. For large aspect ratios $W/W1$, G becomes a strongly decreasing function of frequency and for $W/W1 \simeq 2$, G has approximately a $1/f^2$ dependence (see the dashed curves f in Fig. 1(a),(b)). This suggests that far above resonance, the behavior will be nearly frequency independent [12]. The calculations given in Fig. 3 for the above resonance case confirm this conclusion.

The plots given in Fig. 2 of the susceptance versus frequency are of interest because they show the dependence of the susceptance slope on the aspect ratio $W/W1$. As the aspect ratio decreases the susceptance slope decreases. When the aspect ratio is 1 (hexagonal boundary), the susceptance slope is very close to that estimated for a disk resonator by Fay and Comstock [13] and given by the long-dash curve. As the aspect ratio becomes very small, then the susceptance slope approaches that estimated by Helszajn, James, and Nisbet for the apex coupled triangular resonator [10] and given by the short-dash curve. This result is 1/3 of the Fay, Comstock result.

IV. Conclusions

In this paper it has been demonstrated that linear matrix entry–eigenvalue relations derived from symmetry may be used to greatly reduce the dimensions of the matrices which must be inverted in order to apply the integral equation method to planar two-dimensional problems with symmetry. So, for instance, in the case of six-sided type resonators with threefold symmetry there is a factor of 3 reduction. Furthermore, it was demonstrated that if the Green's function constant C_0 is taken to be 0.5, good accuracy is obtained with only 12 segments around the boundary so that it is possible to program the entire analysis on a desk-top calculator. Calculations of the equivalent admittance of threefold symmetric, 6-sided resonators biased below resonance indicate that an aspect ratio of 0.6 (stripline width to side length) yields a nearly constant conductance shunted by a pure susceptance over greater than one octave bandwidth. The susceptance slope parameter decreases with decreasing aspect ratio in agreement with previous theoretical and experimental results.

References

[1] T. Okoshi, "The planar circuit," in *Rec Professional Groups, IECEJ*, paper SSD68-37/CT68-47, Feb. 17, 1969.

[2] T. Okoshi and T. Miyoshi, "The planar circuit—An approach to microwave integrated circuitry," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 245–252, Apr. 1972.

[3] T. Miyoshi, S. Yamaguchi, and S. Goto, "Ferrite planar circuits in microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 593–600, July 1977.

[4] Y. Ayasli, "Analysis of wide-band stripline circulators by integral equation technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 200–209, Mar. 1980.

[5] T. Okoshi, T. Imai, and K. Ito, "Computer-oriented synthesis of optimum circuit pattern of 3-dB hybrid ring by the planar circuit approach," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 194–202, Mar. 1981.

[6] T. Miyoshi and S. Miyauchi, "The design of planar circulators for wide-band operation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 210–214, Mar. 1980.

[7] H. Bosma, "On stripline Y-circulation at UHF," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, Jan. 1964.

[8] G. P. Riblet, "The measurement of equivalent admittance of 3-port circulators via an automated measurement system," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 401–405, May 1977.

[9] J. Helszajn, *Nonreciprocal Microwave Junctions and Circulators*. New York: Wiley, 1975.

[10] J. Helszajn, D. S. James, and W. T. Nisbet, "Circulators using planar triangular resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 188–193, Feb. 1979.

[11] V. N. Faddeeva, *Computational Methods of Linear Algebra*. New York: Dover, 1959, p. 102.

[12] G. P. Riblet, "The extent of the similarity between below resonance and above resonance operation of standard circulator junctions," in *1978 IEEE MTT-S Int. Microwave Symp. Dig.*, May 1978 pp. 323–325.

[13] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 15–27, Jan. 1965.

[14] J. Matthews and R. L. Walker, *Mathematical Methods of Physics*, 2nd ed. New York: Benjamin 1970, p. 275.

Gordon P. Riblet (M'73) was born in Boston, MA, on December 12, 1943. He received the M.S. and Ph.D. degrees in physics from the University of Pennsylvania, Philadelphia, PA, in 1966 and 1970, respectively.

From 1970 to 1972 he was employed as a Research Scientist at the University of Cologne, Cologne, Germany, performing research in solid-state physics. Since 1972 he has been employed as a Research Scientist at the Microwave Development Laboratories, Natick, MA, working in the areas of ferrite devices and computerized test measurements.

E. R. Bertil Hansson was born in Strömstad, Sweden, on June 20, 1945. He received the M.Sc. and Ph.D. degrees in electrical engineering from Chalmers University of Technology, Gothenburg, Sweden, in 1970 and 1979, respectively.

From 1970 to 1980 he was a Research Assistant at the Division of Network Theory, Chalmers University of Technology. His field of interest at that time was planar microwave ferrite components, in particular junction circulators and phase shifters. In 1979 he received a scholarship from the Sweden–America Foundation for studies in the United States, and is currently with Microwave Development Laboratories (MDL), Inc., Natick, MA. At MDL he has been engaged in theoretical and experimental investigations in the field of computerized test measurements.